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## *Estimating transient heat flows and measuring surface temperatures of a built-up roof*

Charles J. Korhonen

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## PREFACE

This report was prepared by Charles J. Korhonen, Research Civil Engineer, Civil Engineering Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory, while he was at the University of Alaska, Fairbanks, from September 1980 to June 1981. This report was submitted to the author's degree committee as part of the requirement for completion of an advanced degree in Arctic Engineering. The degree committee consisted of John L. Burdick, Head, Department of Civil Engineering; Elbert F. Rice, Professor of Civil Engineering; and John P. Zarling, Head, Department of Mechanical Engineering.

Technical review was provided by S. Flanders and G. Phetteplace of CRREL.

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# ESTIMATING TRANSIENT HEAT FLOWS AND MEASURING SURFACE TEMPERATURES OF A BUILT-UP ROOF

by

Charles J. Korhonen

## INTRODUCTION

To properly size heating and air-conditioning systems for buildings, it is important to know the hourly heat flows through the building's envelope. Because building elements not only resist the flow of heat but store it as well, the maximum heating or cooling demand in the building is not usually in phase with the cyclic changes in ambient temperatures. A sudden increase or decrease in temperatures across a building element will not cause a corresponding change in heat flow through the element. The ASHRAE (1977) Handbook of Fundamentals presents a calculation procedure that an engineer can use to estimate instantaneous rates of heat loss-gain by knowing certain facts about the building envelope and the weather.

My first purpose was to verify this technique by measuring actual heat flows through a model built-up roof with a heat flow sensor. My second purpose was to measure the surface temperatures on that model roof with an infrared scanner and a thermocouple. Since infrared scanners are becoming popular as a tool for diagnosing building temperatures and heat flow problems, a comparison was made between the infrared scanner and thermocouple results.

## EXPERIMENT SETUP

A 4- by 4-ft model built-up roof was designed and built to serve as the building component on which to evaluate the ASHRAE (1977) Handbook of Fundamentals computation technique and on which to measure surface temperatures. The model roof was placed on an electrically heated box that was set on the roof of the Duckering Building on the Fairbanks campus of the University of Alaska. Each layer of the model roof was carefully dimensioned and assembled to provide a building component of known cross

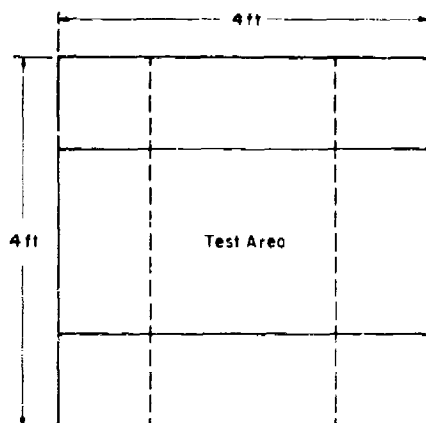


Figure 1. Cross section of built-up roof.

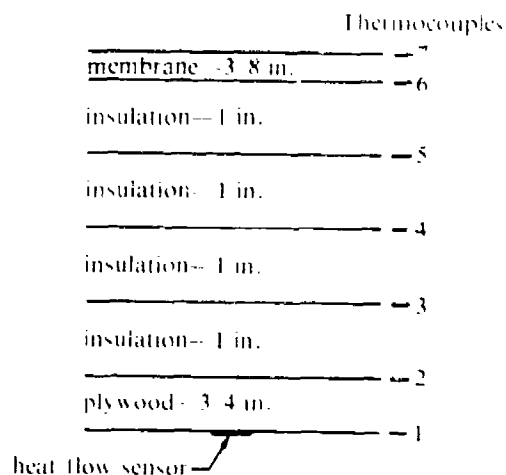


Figure 2. Insulation pattern in built-up roof.

section. Thermocouples, placed between each layer, and a heat flow sensor, taped to the underside of the roof, were read hourly from 3 a.m. to 12 midnight on 24 April 1981 to furnish data for evaluating the Handbook of Fundamentals technique (Appendix A). A thermocouple was embedded just below a film of roof cement to measure surface temperatures.

Figure 1 shows the arrangement of the heat flow sensor and copper-constantan thermocouples. The insulation in the roof consisted of four 1-in. layers of perlite boardstock, which is a blend of perlite particles and reprocessed cellulose fibers. A 3/4-in. plywood sheet served as the deck. The built-up membrane consisted of alternate layers of asbestos cement that was applied cold and three plies of no. 15 roofing felt. No gravel was applied to the roof surface. (As a note of interest, although care was taken to exclude all voids during the construction of the built-up membrane, a few small blisters yet developed in the membrane. It is easy to understand how blisters can be built into actual roofs.)

Figure 2 shows the layout of the 2-ft-wide insulation boards in the 4-ft-wide roof. This pattern created a 2- x 2-ft test area for measuring heat flows that was free of insulation seams.

Five 60-W light bulbs connected to a thermostat provided heat whenever the box temperature dropped below 75°F. The inside air was continually mixed by a small electric fan. The heat flow sensor was shielded from the light bulbs by a sheet of plywood.

## RESULTS

### Transfer function

The transfer function method is presented in Handbook of Fundamentals (ASHRAE 1977) for calculating the heat conducted through multilayered building elements exposed to transient thermal conditions. The Handbook recommends that "sol-air" temperatures be used in carrying out the calculations, but as I will show, the use of roof surface temperatures yields similar results. Thus, it might be possible to use an infrared scanner to remotely gather these data.

The sol-air temperature can be represented by the following equation:

$$T_e = T_o + \alpha I_t / h_o - \epsilon \Delta R / h_o \quad (1)$$

where  $T_e$  = sol-air temperature ( $^{\circ}\text{F}$ )

$T_o$  = outdoor air temperature ( $^{\circ}\text{F}$ )

$\alpha$  = solar absorptance

$I_t$  = incident solar radiation ( $\text{Btu/hr}\cdot\text{ft}^2$ )

$h_o$  = air film heat transfer coefficient ( $\text{Btu/hr}\cdot\text{ft}^2\cdot^{\circ}\text{F}$ )

$\epsilon$  = surface emissivity

$\Delta R$  = difference between the longwave radiation incident on the surface from the sky and surroundings, and the radiation emitted by a black body at outdoor air temperature ( $\text{Btu/hr}\cdot\text{ft}^2$ ).

An estimate of the solar radiation incident on the test roof,  $I_t$ , is given for latitude 64 degrees north in Table 25 of the Handbook of Fundamentals (ASHRAE 1977, Chapter 26). The listed values cannot be used directly because they represent the solar heat gain through double-strength sheet glass. Consequently they must first be multiplied by 1.15 (ASHRAE 1977) to correct for the solar energy excluded by the glass. Since Fairbanks was on daylight savings time during this experiment, the tabulated data were shifted 1 hour. The corrected and shifted values of  $I_t$  from the Handbook are listed in Table 1. These values increase from a very small thermal load in the morning to a maximum at 1 p.m. (solar noon) and then decrease in like manner in the afternoon. The day on which this experiment was run was nearly clear so no corrections were made for cloud cover.

Table 2. Sol-air and test roof surface temperatures.

		Time	Sol-air (°F)	Roof surface (°F)
Table 1. Solar radiation incident on test roof (clear day).		1 a.m.	31.2	31.8
		2	29.4	27.2
		3	27.6	22.6
		4	29.2	24.8
		5	22.0	29.0
		6	31.9	36.2
		7	44.1	59.0
		8	55.2	66.8
		9	76.5	109.8
		10	89.4	111.4
		11	100.9	121.2
		Noon	112.1	121.2
		1 p.m.	111.0	103.4
		2	110.9	109.4
		3	103.7	82.4
		4	92.4	73.0
		5	81.3	66.4
		6	69.4	60.8
		7	52.7	46.6
		8	42.1	39.0
		9	39.6	35.6
		10	39.0	36.0
		11	38.2	37.8
		12	33.0	36.4
Local time (hr)	I (Btu/hr·ft <sup>2</sup> )			
6 a.m.	2.3			
7	24.2			
8	58.7			
9	97.8			
10	133.4			
11	161.0			
Noon	178.3			
1 p.m.	184.0			
2	178.3			
3	161.0			
4	133.4			
5	97.8			
6	58.7			
7	24.3			
8	2.3			

The Handbook indicates that a flat roof surface loses more longwave radiation than it gains from the surroundings. As a consequence,  $\epsilon \Delta R/h_o$  equals  $-7^\circ\text{F}$ .

By measuring air temperatures and selecting  $\epsilon$  and  $h_o$  values, the engineer can use eq 1 to calculate sol-air temperatures. For example, the sol-air temperature at 10 a.m. on 24 April with  $\epsilon = 0.93$  and  $h_o = 3$  is

$$T_e = 55^* + \frac{0.93}{3} (133.4) - 7$$

$$T_e = 89.4^\circ\text{F}.$$

The roof surface temperature (Table 2) at that same hour was 111.4°F. Although this is not good agreement, the sol-air and roof temperatures were

\* The air temperatures are taken from Appendix A.

Table 3. Heat flows (Btu/hr) through test roof calculated with eq 2 using sol-air temperatures.

Time (hr)	First iteration	Second iteration	Third iteration	Fourth iteration
1	-0.9696940	-2.4442700	-2.4442710	-2.4442710
2	-1.8340240	-2.7049590	-2.7049600	-2.7049600
3	-2.4379300	-2.9298360	-2.9298360	-2.9298360
4	-2.8442920	-3.1179190	-3.1179190	-3.1179190
5	-3.1046800	-3.2560730	-3.2560730	-3.2560730
6	-3.2949180	-3.3785220	-3.3785220	-3.3785220
7	-3.4269170	-3.4730540	-3.4730540	-3.4730540
8	-3.2887370	-3.3141920	-3.3141920	-3.3141920
9	-2.8673470	-2.8813900	-2.8813900	-2.8813900
10	-2.1868970	-2.1946430	-2.1946430	-2.1946430
11	-1.2659210	-1.2701940	-1.2701940	-1.2701940
12	-0.2885270	-0.2908844	-0.2908843	-0.2908843
13	0.6516642	0.6503637	0.6503637	0.6503637
14	1.4486700	1.4479530	1.4479530	1.4479530
15	1.9694940	1.9690980	1.9690980	1.9690980
16	2.2109110	2.2106920	2.2106920	2.2106920
17	2.1383280	2.1382070	2.1382070	2.1382070
18	1.7770200	1.7769540	1.7769540	1.7769540
19	1.2124580	1.2124210	1.2124210	1.2124210
20	0.4780363	0.4780160	0.4780160	0.4780160
21	-0.3839333	-0.3839445	-0.3839445	-0.3839445
22	-1.1876650	-1.1876710	-1.1876710	-1.1876710
23	-1.7802520	-1.7802560	-1.7802560	-1.7802560
24	-2.1646280	-2.1646300	-2.1646300	-2.1646300

closer during the evening hours. Since the sol-air temperatures are based on estimated incoming solar radiation, and roof surface temperatures reflect actual conditions. I feel that the thermocouple readings will yield the best results in the transfer function calculations.

Equation 2, which is recommended by the Handbook for estimating hourly heat flows through a building element, was used to calculate the heat flow values in Tables 3 and 4. The sol-air and surface temperature data in Table 2 simulated the outside air temperatures. Inside air temperatures were held at 75°F, while the indoor and outdoor air film coefficients were the same as those used in eq 1.

$$q_{e,\tau} = A \left[ \sum_{n=0} b_n (t_{e,\tau} - n\Delta) - \sum_{n=1} d_n \left( \frac{q_{e,\tau} - n\Delta}{A} \right) - t_{rc} \sum_{n=0} c_n \right] \quad (2)$$

where  $A$  = indoor surface area (ft<sup>2</sup>)

$q_{e,\tau}$  = heat flow through indoor surface (Btu/hr)

Table 4. Heat flows (Btu/hr) through test roof calculated with eq 2 using actual roof temperatures.

Time (hr)	First iteration	Second iteration	Third iteration	Fourth iteration
1	-0.9573820	-2.6242310	-2.6242320	-2.6242320
2	-1.7733140	-2.7543960	-2.7543960	-2.7543960
3	-2.3911340	-2.9446140	-2.9446140	-2.9446140
4	-2.8852500	-3.1930050	-3.1930050	-3.1930050
5	-3.2545280	-3.4247800	-3.4247800	-3.4247800
6	-3.4288710	-3.5228840	-3.5228840	-3.5228840
7	-3.3901210	-3.4420010	-3.4420010	-3.4420010
8	-3.0390340	-3.0676570	-3.0676570	-3.0676570
9	-2.3527220	-2.3685120	-2.3685130	-2.3685130
10	-1.3356600	-1.3443700	-1.3443700	-1.3443700
11	0.0230660	0.0182607	0.0182607	0.0182607
12	1.1916790	1.1890280	1.1890280	1.1890280
13	2.0744340	2.0729720	2.0729720	2.0729720
14	2.5047830	2.5039760	2.5039760	2.5039760
15	2.5186090	2.5181640	2.5181640	2.5181640
16	2.2870190	2.2867740	2.2867740	2.2867740
17	1.6627210	1.6625860	1.6625860	1.6625860
18	0.9325699	0.9324952	0.9324952	0.9324952
19	0.2697038	0.2696626	0.2696626	0.2696626
20	-0.3697135	-0.3697362	-0.3697362	-0.3697362
21	-1.0655870	-1.0655990	-1.0655990	-1.0655990
22	-1.7079740	-1.7079810	-1.7079810	-1.7079810
23	-2.1906630	-2.1906670	-2.1906670	-2.1906670
24	-2.4769670	-2.4769690	-2.4769690	-2.4769690

$\tau$  = time

$\Delta$  = time interval

$n$  = number of transfer function coefficients

$t_{e,\tau} - n\Delta$  = sol-air or roof temperature at time  $\tau - n\Delta$  ( $^{\circ}\text{F}$ )

$t_{rc}$  = indoor temperature ( $^{\circ}\text{F}$ )

$b_n, d_n, c_n$  = transfer function coefficients.

The transfer function coefficients ( $b_n, d_n, c_n$ ) are listed for typical wall and roof constructions in the Handbook. Alternatively, the engineer can use Mitalas and Arseneault's (1972) computer program to calculate transfer function coefficients, as I did for this report. The University of Alaska has this program on file in its computer. The computer calculated the transfer function coefficients after I supplied it with an outdoor film coefficient  $h_o = 3.0 \text{ Btu/hr}\cdot\text{ft}^2\cdot^{\circ}\text{F}$  and an indoor film coefficient  $h_i = 1.46 \text{ Btu/hr}\cdot\text{ft}^2\cdot^{\circ}\text{F}$ , and the density, conductivity,

specific heat, thickness, emissivity and absorptivity of the roof.

The resulting transfer function coefficients for the test roof are:

$$\begin{array}{lll} b_0 = 0.00018 & & d_0 = 1.00000 \\ b_1 = 0.00795 & & d_1 = -0.79820 \\ b_2 = 0.01409 & \Sigma c_n = 0.02526 & d_2 = 0.14593 \\ b_3 = 0.00297 & n=0 & d_3 = -0.00552 \\ b_4 = 0.00007 & & d_4 = 0.00003 \end{array}$$

Inspection of eq 2 reveals that the calculation of heat flow for any given hour requires temperatures at that hour and preceding hours, plus heat flows at preceding hours. Since, initially, heat flows are not known, any value of heat flow can be used as a first guess. Through an iterative process the actual calculated heat flow will evolve.

Equation 2 is composed of three sections. The first section sums the product of each b coefficient and simulated outdoor condition. The present outdoor temperature, either sol-air or roof surface, is multiplied by the  $b_0$  coefficient. Next the outdoor temperature from the previous hour is multiplied by the  $b_1$  coefficient. This continues until all the b coefficients are used and then the products are summed to complete the first section of eq 2.

The second section is the summation of the d coefficients multiplied by the previous values of heat flow. Equation 2 does not use  $d_0$  in its calculations. The  $d_1$  coefficient is multiplied by the heat flow calculated for the previous hour, then  $d_2$  is multiplied by the heat flow 2 hours back in time. This continues until all the d coefficients (except  $d_0$ ) are used. The products are then summed to complete the second section of eq 2. The third section is calculated only once since there is only one c coefficient and the inside temperature is constant.

Once all three sections are calculated they are combined to give the values of heat flows for each hour of the day. This process is repeated until the latest calculated heat flow closely agrees with the previous iteration. This convergence of values is shown in Tables 3 and 4. In this case either the third or fourth iteration yields the desired estimated heat flow. Appendix B lists the BASIC computer program written to perform the calculations.

### Heat flow and temperature measurements

Copper-constantan thermocouples were used for this experiment. A Fluke Model 2175A Digital Thermometer was used to measure the thermocouple readings. It maintained its own internal reference and automatically converted the millivolt signal from the thermocouples into a temperature in degrees Fahrenheit.

The heat flow sensor used in this experiment consisted of a number of thermocouples embedded in a material of known thermal conductivity. Half of the thermocouples were embedded near the top surface and half near the bottom surface of the sensor to measure the temperature change across the sensor. This produced a millivolt reading proportional to heat flow. This heat flow sensor had a calibration constant of 93.1 mV, equal to 500 Btu/hr-ft<sup>2</sup>. A Keithley Instruments Model 155 Microvoltmeter was used to obtain heat flow sensor readings. Data obtained from the heat flow and temperature sensors are presented in Appendix A.

### Infrared temperature measurements

I used an infrared camera to determine how accurately surface temperatures could be measured. An AGA Thermovision 750, which measured radiant energy within a wavelength band of 2 to 5.6  $\mu$ m, was used for these measurements.

The energy emitted from the surface of a body is a function of the object's temperature and emissivity. Emissivity relates to the ease with which an object emits infrared radiation. Not all surfaces emit the same amount of energy when they are at identical temperatures. A surface that emits the maximum amount of energy has an emissivity of 1. All objects in nature possess emissivities less than 1. Therefore, an accurate measurement of surface temperature requires a knowledge of the longwave emissivity of the object to correct for varying radiation potentials.

Equation 3 accounts for both radiation emitted and reflected from the surface of an object. (This emissivity equation is presented in the owner's manual [AGA Infrared Systems 1973] for the infrared camera.)

$$I_o = \frac{\Delta I_{or}}{\epsilon_o} + \frac{\epsilon_r}{\epsilon_o} I_r + \left(1 - \frac{\epsilon_r}{\epsilon_o}\right) I_a \quad (3)$$

where  $\Delta i_{or}$  = image isotherm difference ( $i_o - i_r$ ) read from display unit

$\epsilon_o$  = object emissivity (roof)

$\epsilon_r$  = reference emissivity (timber)

$I_r$  = absolute isotherm read from calibration chart (Fig. 3)  
knowing reference temperatures  $T_r$

$I_a$  = absolute isotherm read from calibration chart (Fig. 3)  
knowing ambient temperature  $T_a$ .

By knowing the temperature and emissivity of a reference object and the air temperature and the emissivity of another object we can use eq 3 with Figure 3 to calculate the unknown temperature of the second object.

I measured radiation with an old wooden timber of an assumed emissivity of 0.9 in the same field of view as the model roof. One thermocouple monitored the timber's surface temperature while a second measured the air temperature. (The air temperature thermocouple was shaded from the sun.) The roof was assumed to have a high emissivity of 0.93 since it was fresh, clean asphalt.

During one measurement cycle the timber and air temperatures were  $44.6^\circ\text{F}$  and  $46.9^\circ\text{F}$  respectively. The resulting absolute isotherm levels  $I_r$  and  $I_a$  as read from a calibration chart (Fig. 3) were 12.8 and 13.4 respectively. The isotherm markers of the timber and the roof, read from the camera display unit, were 0.5 and 0.4 respectively. The image isotherm levels  $i_r$  and  $i_o$  were then obtained by multiplying the isotherm markers

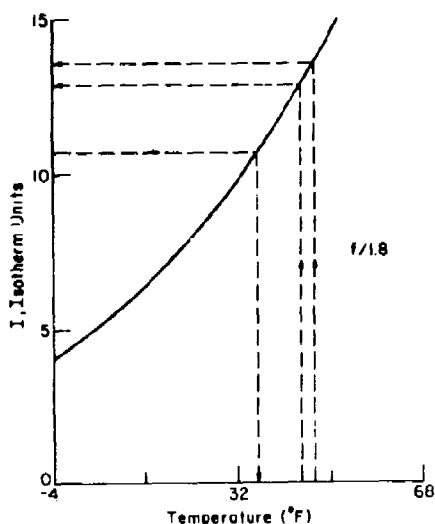


Figure 3. Calibration chart. f/1.8 denotes aperture setting of camera.

Table 5. Surface temperature of test roof as measured by a thermocouple and an infrared camera on 24 April 1981.

Measurement number	Thermocouple no. 7 (°F)	I.R. camera (°F)	Time	Conditions
1	112.2	111.2	10:30 a.m.	Sunny
2	113.0	106.7	12:50 p.m.	Partly cloudy
3	91.4	83.8	1:15 p.m.	Cloudy
4	102.6	95.9	1:20 p.m.	Sunny
5	105.6	98.6	1:22 p.m.	Sunny
6	108.2	102.4	1:25 p.m.	Sunny
7	37.8	35.2	9:30 p.m.	Clear
8	37.6	36.5	9:40 p.m.	Clear
9	37.4	35.6	9:45 p.m.	Clear
10	37.6	36.5	9:50 p.m.	Clear
11	37.9	36.9	11:30 p.m.	Clear

by the range setting of the camera, which was 20 during this measurement cycle;  $i_r$  and  $i_o$  became 10 and 8 respectively.  $I_o$  was then determined by inserting the above values into eq 3:

$$I_o = \frac{8 - 10}{0.93} + \frac{0.90}{0.93} (12.8) + \left(1 - \frac{0.90}{0.93}\right) 13.4$$

$$I_o = 10.7.$$

The roof surface temperature taken from Figure 3 was 36.9°F. Thermocouple 7 (Fig. 1) measured the roof surface temperature to be 37.9°F.

I repeated this procedure a number of times; the results are shown in Table 5. The greatest discrepancy between the roof temperatures measured with the thermocouple and those measured with the infrared camera occurred during the daytime. Measurement 3 in Table 5 was 7.6°F below that of its corresponding thermocouple measurement. The evening radiation measurements were all within 3°F of the thermocouple temperatures. The best correlation occurred later at night. In all cases the radiation measurements fell somewhat below the thermocouple measurements. Although Table 5 does not show complete agreement between the two measurement techniques, it does show that surface temperatures can be approximated with an infrared camera, especially during the night.

#### Uncertainty analysis

An area of uncertainty in determining unknown temperatures from radia-

tion measurement lies in how accurate the values substituted into eq 3 are. Holman (1971) describes a method for specifying the uncertainty in the results of experimental data. His analysis will be followed to determine the uncertainty in the results of eq 3.

The result,  $I_o$ , of eq 3 is a function of the independent variables  $\Delta i_{or}$ ,  $\epsilon_o$ ,  $\epsilon_r$ ,  $I_r$  and  $I_a$ ; thus

$$I_o = f(\Delta i_{or}, \epsilon_o, \epsilon_r, I_r, I_a). \quad (4)$$

If  $W_r$  represents the uncertainty of  $I_o$  then  $W_1$ ,  $W_2$ ,  $W_3$ ,  $W_4$  and  $W_5$  represent the uncertainty or range of possible values of the independent variables. By specifying this range of possible values for each independent variable, we can determine the uncertainty in  $I_o$  as shown below:

$$W_r = \left[ \left( \frac{\partial I_o}{\partial \Delta i_{or}} W_1 \right)^2 + \left( \frac{\partial I_o}{\partial \epsilon_o} W_2 \right)^2 + \left( \frac{\partial I_o}{\partial \epsilon_r} W_3 \right)^2 + \left( \frac{\partial I_o}{\partial I_r} W_4 \right)^2 + \left( \frac{\partial I_o}{\partial I_a} W_5 \right)^2 \right]^{1/2}. \quad (5)$$

The independent variables have the following ranges:  $\epsilon_o = 0.93 \pm 0.02$ ,  $\epsilon_r = 0.90 \pm 0.02$ ,  $\epsilon_a = 1.0 \pm 0.02$ ,  $I_r = 12.8 \pm 0.20$ ,  $I_a = 13.4 \pm 0.20$ . Although  $I_r$  and  $I_a$  are functions of temperature, the chart from which  $I_r$  and  $I_a$  are taken controls the uncertainty in these values. I feel that  $I_r$  and  $I_a$  can only be read within  $\pm 0.2$  of their actual values as shown above.  $\Delta i_{or}$  is a function of how well the isotherm marker can be read from the display unit of the camera. Due to the flicker of the cathode-ray tube, I do not feel that I could consistently read the isotherm marker closer than  $\pm 0.02$ .

Using eq 5 along with the stated ranges of the independent variable gives

$$\begin{aligned} \partial I_o / \partial \Delta i_{or} &= 1 / \epsilon_o = 1 / 0.93 = \underline{1.08} \\ \partial I_o / \partial \epsilon_o &= -\Delta i_{or} / \epsilon_o^2 - \epsilon_r I_r / \epsilon_o^2 + \epsilon_r I_a / \epsilon_o^2 \\ &= -2 / 0.93^2 - 0.90(12.8) / 0.93^2 + 0.90(13.4) / 0.93^2 \\ &= \underline{2.94} \end{aligned}$$

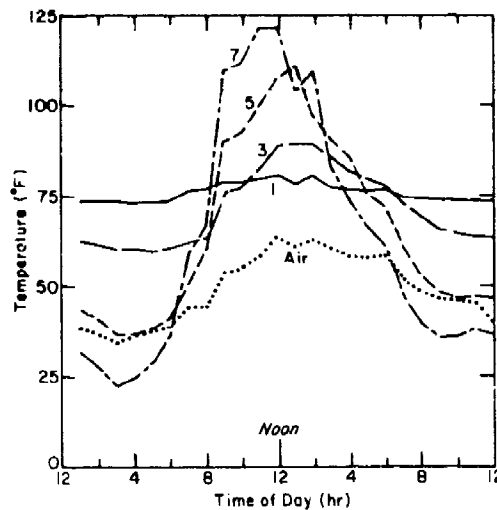


Figure 4. Thermocouple temperatures.

$$\partial I_o / \partial \epsilon_r = I_r / \epsilon_o - I_a / \epsilon_o$$

$$= 12.8 / 0.93 - 13.4 / 0.93$$

$$= \underline{-0.65}$$

$$\partial I_o / \partial I_r = \epsilon_r / \epsilon_o = 0.90 / 0.93 = \underline{0.97}$$

$$\partial I_o / \partial I_a = 1 - \epsilon_r / \epsilon_o = 1 - 0.90 / 0.93 = \underline{0.03}$$

The uncertainty in measuring  $I_o$  (eq 3) becomes

$$W_r = [(1.08 \times 0.02)^2 + (2.94 \times 0.02)^2 + (-0.65 \times 0.02)^2$$

$$+ (0.97 \times 0.2)^2 + (0.03 \times 0.2)^2]^{1/2}$$

$$W_r = 0.20$$

Thus the  $I_o$  reading of 10.7 from the Infrared Temperature Measurement section obtained with the infrared camera is accurate to  $\pm 0.20$  unit.

#### Evaluation of transfer function

The transient nature of the heat flow through the test roof is shown by the graph of thermocouple temperatures in Figure 4 (values taken from Appendix A). The effect of a maximum surface temperature occurring at 11

Table 6. Heat flow through test roof (Btu/hr).  
A negative sign indicates heat being lost  
to the outdoors.

Time	Transfer function (based on roof temperatures)	Heat flow sensor
1 a.m.	-2.6	-3.9
2	-2.8	-4.1
3	-2.9	-4.3
4	-3.2	-4.4
5	-3.4	-4.5
6	-3.5	-4.7
7	-3.4	-5.9
8	-3.1	-4.4
9	-2.4	-3.2
10	-1.3	-3.2
11	0.0	1.2
Noon	1.2	3.0
1 p.m.	2.1	4.3
2	2.5	4.2
3	2.5	3.3
4	2.3	2.1
5	1.7	1.6
6	0.9	-0.5
7	0.3	-0.9
8	-0.4	-1.1
9	-1.1	-3.0
10	-1.7	-3.4
11	-2.2	-3.7
12	-2.5	-3.8

a.m. is not sensed by the underside of the roof until about 2 p.m. This should come as no surprise when the heat storage effects of the insulation are considered. We also see a maximum heat flow at the inside roof surface at 2 p.m.

Does the transfer function predict the same thermal lags shown in Figure 4? Table 6 compares heat flow through the roof as computed with the transfer function to that measured with the heat flow sensor. (Roof surface temperatures, which are actual values, were used to derive the transfer function values as opposed to using the estimated sol-air temperatures.) The heat flow sensor data were taken from Appendix A. Generally, the transfer function predicts the same thermal lag as shown in Figure 4. Also, the periods of heat loss and gain correspond for the results from the transfer function and the heat flow sensor. Both measurement techniques show heat being lost from the roof until 11 a.m., then the roof gaining

heat until 6 or 7 p.m., whereafter it again began to lose heat. It appears that the transfer function is able to model the time and direction of heat flow.

However, the magnitude of heat flow in Table 6 differs considerably between the transfer function results and the heat flow sensor results. Because of the short measurement period of this experiment, it is difficult to assess this discrepancy other than to explain some of the potential sources of error.

In reading the microvoltmeter it was often necessary to take an average of several readings to get one hourly reading because the meter needle often swung back and forth. At first I thought the cycling on and off of the lights to be the probable cause of the apparent jumps in the heat flow readings, but careful control of their cycling did nothing to dampen the swing of the needle. Even turning the fan off did not help. Consequently, I used a system of averaging several readings. Perhaps some other sort of recorder, such as a stripchart recorder, would have yielded better results.

#### SUMMARY AND CONCLUSIONS

I used a computer method that was modified to employ roof surface temperatures and heat storage capacities of materials to estimate the hour-by-hour transient heat flow through a known cross section of built-up roof. Thermocouples and a heat flow sensor provided a check. Also, I compared roof surface temperatures measured with an infrared camera to roof surface temperatures measured with a thermocouple.

The computer method predicted the time and the direction of heat flow through the built-up roof but the magnitude of those heat flows did not correspond well to heat flows measured by the heat flow sensor. This could be due to the insensitivity of the heat flow sensor and to the instability of the microvoltmeter needle when readings were taken. The accuracy of the transfer function should be further checked by use of better equipment and longer measurement cycles.

Surface temperature measurements taken with the infrared camera corresponded fairly well with the roof surface temperatures recorded by a thermocouple. I found the best results at night when there was no solar influence.

#### LITERATURE CITED

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# APPENDIX A: THERMOCOUPLE AND HEAT FLOW READINGS

Thermocouple readings in °F; heat flow readings in mV; negative sign indicates heat flowing from inside surface of roof up into insulation.

TC	*1 a.m.	*2 a.m.	3 a.m.	4 a.m.	5 a.m.	6 a.m.
1	73.5	73.5	73.6	73.2	73.2	74.0
2	71.5	71.5	71.6	71.0	70.6	70.8
3	62.5	61.5	60.6	60.2	59.8	60.2
4	53.5	51.5	49.4	49.0	49.0	50.6
5	43.5	40.1	36.8	37.0	38.4	41.0
6	33.1	28.7	24.4	25.8	29.4	35.2
7	31.8	27.2	22.6	24.8	29.0	36.2
Air	38.2	36.4	34.6	36.2	37.6	28.2
Heat flow	-0.73	-0.77	-0.80	-0.82	-0.84	-0.87

TC	7 a.m.	8 a.m.	9 a.m.	10 a.m.	11 a.m.	Noon
1	76.2	76.6	78.6	78.6	79.8	80.2
2	71.6	70.0	74.8	75.6	77.4	80.6
3	62.0	63.2	75.8	77.2	82.6	88.4
4	54.6	59.6	79.8	82.4	88.8	96.6
5	51.0	60.4	90.0	92.4	100.0	107.4
6	57.4	64.8	107.8	110.4	119.8	121.8
7	59.0	66.8	109.8	111.4	121.2	121.2
Air	43.6	44.0	53.2	55.0	58.0	63.8
Heat flow	-1.1	-0.82	-0.60	-0.60	0.24	0.56

TC	1 p.m.	2 p.m.	3 p.m.	4 p.m.	5 p.m.	6 p.m.
1	78.0	80.4	77.4	76.4	76.2	76.2
2	80.2	80.2	79.2	78.0	77.2	75.2
3	89.2	89.0	85.2	81.4	79.4	77.2
4	98.8	95.4	89.6	84.6	79.6	76.3
5	111.0	97.8	90.6	85.4	76.0	71.9
6	116.4	105.0	83.2	79.0	68.4	63.2
7	103.4	109.4	82.4	73.0	66.4	60.8
Air	61.0	62.6	60.8	58.0	58.0	58.2
Heat flow	0.80	0.79	0.62	0.40	0.30	-0.10

TC	7 p.m.	8 p.m.	9 p.m.	10 p.m.	11 p.m.	Midnight
1	74.8	74.4	73.8	73.8	73.6	73.4
2	75.0	74.0	72.4	72.2	71.8	71.4
3	72.8	69.0	65.4	64.8	63.8	63.4
4	69.0	62.8	57.8	57.0	56.0	55.6
5	61.2	53.0	48.0	47.0	47.4	46.8
6	48.4	40.4	36.8	36.6	38.8	37.4
7	46.6	39.0	35.6	36.0	37.8	36.4
Air	52.2	48.4	46.6	46.0	45.2	40.0
Heat flow	-0.17	-0.20	-0.56	-0.63	-0.68	-0.70

\*Readings derived by prorating the midnight and 3 a.m. readings.

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1 PRINT"***** INFORMATION *****"
2 PRINT
3 PRINT      "THIS PROGRAM CALCULATES THE HOURLY TRANSIENT"
4 PRINT      "HEAT FLOW THRU A MULTILAYERED BUILDING COMPONENT"
5 PRINT      "BY USING THE <TRANSFER FUNCTION METHOD>"
6 PRINT      "PRESENTED BY ASHRAE, HANDBOOK OF FUNDAMENTALS (1977)."

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180 PRINT "THE FOLLOWING ARE THE SURFACE TEMPERATURES USED IN THIS PROGRAM"
181 PRINT
182 PRINT "          HOUR          TEMP"
183 PRINT
190 FOR N=I1 TO (I1+I-1)
191 S1=N-I1+1
200 PRINT S1,T(N)
210 NEXT N
220 FOR M=1 TO (I1-1)
230 L=M+I
240 T(M)=T(L)
250 NEXT M
277 PRINT
278 PRINT "THE B-COEFFICIENTS ARE"
279 PRINT
280 FOR J=1 TO I1
301 PRINT USING 302,J,B(J)
302:      ***      **.*#####
303 NEXT J
332 FOR U1=1 TO I2
333 M1=U1-1
334 D(M1)=D(U1)
335 NEXT U1
337 PRINT
338 PRINT "THE D-COEFFICIENTS ARE"
339 PRINT
340 FOR K=0 TO (I2-1)
361 PRINT USING 362,K,D(K)
362:      ***      **.*#####
370 NEXT K
371 PRINT
372 PRINT "          HOUR          HEATFLOW BTU/HR"
373 PRINT
410 FOR A=1 TO 4
420 FOR P=I1 TO (I+I1-1)
430 X=P-(I1-1)
450 FOR S=1 TO I1
460 P1=(P+1)-S
470 T1(S)=T(P1)
480 NEXT S
490 B2=0
500 FOR U=1 TO I1
510 B=B(U)*T1(U)
520 B2=B2+B
530 NEXT U
540 Q(0)=0
550 D2=0
560 FOR V=1 TO (I2-1)
570 V1=P-V
600 D=D(V)*Q1(V1)
610 D2=D2+D
620 NEXT V

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630 Q(X)=R2-D2-(T2*C)
640 PRINT X,Q(X)
650 FOR G=I2 TO (I2+I-1)
660 H=G-(I2-1)
670 Q1(G)=Q(H)
680 NEXT G
690 FOR L=1 TO (I2-1)
700 W=L+I
710 Q1(L)=Q1(W)
720 NEXT L
721 NEXT F
722 PRINT
730 NEXT A
800 END

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